JOURNAL OF MATERIALS SCIENCE **38** (2003) 1081 – 1085

A Golden Section approach to optimization of automotive friction materials

Y. LU
College of Materials Science and Engineering, Beijing University of Chemical Technology, Beijing 100029, P.R. China
E-mail: yafeilu@siu.edu

A Golden Section approach combined with relational grade analysis is proposed as an experimental design tool helpful in the development of new automotive friction materials. Golden Section was used to design the volume fraction of the components systematically. The changes in friction performance (friction coefficient and wear) measured using Friction Assessment and Screening Test (FAST) can be correlated with component variations by use of relational grade analysis. This approach was utilized to optimize a non-metallic friction material containing seven ingredients including two fibers (aramid and slag fiber), four fillers (Al_2O_3, $BaSO_4$, graphite and nitrile rubber) and one binder (benzoxazine). The volume fraction of seven components was varied simultaneously in order to optimize the parameters of friction coefficient and wear with a minimum number of tests. Three phases were performed to find the optimal proportion of the components. The optimized friction performance was obtained after doing 19 formulation experiments.

1. Introduction

Automotive friction materials are composed of many components. The interactions among the components are complex so that friction material formulations are usually achieved through trial and error, coupled with prior experience and testing expertise [1–3]. For a multiple components system optimizing by changing one variable at a time experimentation requires many experiments and cannot guarantee finding an optimizing formulation. It has been proved that the friction performance of commercial brake pad formulations currently used in the market is not optimized [4]. There are three main problems in the development of friction materials. One is how to select the raw materials. The second is how to find the interactions among the components. The third is how to optimize the formulations. Recently, some efforts in relating to these problems were done. Combinatorial approach to screening of raw materials [5], intelligent selection of raw materials [6], Taguchi design of formulation experiments [7], uniform design for formulations [8], application of chemometrics to friction formulations [9] and friction materials design using database [10] have been developed. However, due to the complexity of the multiple component friction materials the problems are not yet solved.

The relationship between composition and friction performance of automotive friction materials can be mathematically described by a set form as:

$$f\colon \left\{ \sum M_i V_i \right\} \to P \tag{1}$$

where $f\colon \{\sum M_i V_i\}$ represents friction material formulations. P is a set of properties of friction materials and $P = \{P_1, P_2, P_3, \ldots\}$ where $P_1 =$ friction coefficient (μ), $P_2 =$ wear %, $P_3 =$ surface roughness, etc. M is a set of the raw materials used in the formulations. Automotive friction materials consist of fibers (F), fillers (f) and binders (B), thus, M can be expressed by a set form as:

$$M = FfB \tag{2}$$

where $F = \{F_1, F_2, F_3, \ldots\}$, $f = \{f_1, f_2, f_3, \ldots\}$ and $B = \{B_1, B_2, B_3, \ldots\}$. V is volume fraction of M and $V = \{V_1, V_2, V_3, \ldots\}$ with $\sum V_i = 1$. From Equation 1 it is clear that to obtain the best performance (P), an optimal proportion (V) of best raw materials (M) should be chosen.

In this study a new non-metallic friction material containing aramid pulp, slag fiber, Al_2O_3, $BaSO_4$, graphite, NBR (nitrile rubber) and benzoxazine (ring-open polymerized phenolic) was developed using an optimizing formulation technique which is based on Golden Section and relational grade analysis.

2. Theoretical background

2.1. Golden Section principle and Golden Section sequence

Golden Section has been utilized to be an optimization approach in solving actual scientific, technical and engineering problems. Golden Section is a number (0.618) which satisfies the equation $r^2 = 1 - r$ [10]. If a line segment be divided into two parts, the most pleasing division is when the left-hand portion is of length $a = 1$ and the right-hand portion is of length $b = 0.618$. If the total length $(a + b)$ is 1, then one portion should be of length 0.618 by the principle of dynamic symmetry [11, 12].

The line can be continuously divided and Golden Section sequence (0.618^n) can be obtained as follow [13]:

	Left Line	Right Line
________________	$0.618^0 = 1$	
_________._______	$0.618^1 = 0.618$	$0.618^2 = 0.382$
_________.______	$0.618^2 = 0.382$	$0.618^3 = 0.236$
________._____	$0.618^3 = 0.236$	$0.618^4 = 0.146$
_______._____	$0.618^4 = 0.146$	$0.618^5 = 0.09$
______.___	$0.618^5 = 0.09$	$0.618^6 = 0.056$
_____.___	$0.618^6 = 0.056$	$0.618^7 = 0.034$
_____.__	$0.618^7 = 0.034$	$0.618^8 = 0.022$
____._	$0.618^8 = 0.022$	$0.618^9 = 0.014$

...

2.2. Multiple components permutation by means of Golden Section principle

All possible formulations for the components composed of fibers ($V_F = 0.382$), fillers ($V_f = 0.382$) and binder ($V_b = 0.236$) groups can be obtained by permutation governed by Golden Section principle as follow:

One fiber and one filler

V_{F1}	V_{f1}	V_B
0.382	0.382	0.236

One fiber and two fillers or two fibers and one filler

V_{F1}	V_{f1} V_{f2}	V_B
0.382	0.236, 0.146	0.236
0.382	0.146, 0.236	0.236

Two fibers and two fillers

V_{F1} V_{F2}	V_{f1} V_{f2}	V_B
0.236, 0.146	0.236, 0.146	0.236
0.236, 0.146	0.146, 0.236	0.236
0.146, 0.236	0.236, 0.146	0.236
0.146, 0.236	0.146, 0.236	0.236

and so on.

2.3. Relational grade analysis

Relational grade analysis is one of the important fields of gray system theory [14–17]. Here the relative relational grade analysis was used to establish the relationship between formulation composition and friction performance and find which ingredient has better or bad effect on the performance. It needs to set up normalized composition matrix $x(k)$ and normalized performance matrix $y(k)$, and calculate relational coefficient $\xi_i(k)$ and relation degree γ_i:

$$\xi_i(k) = \frac{\min\min|y_i(k) - x_i(k)| + 0.5\max\max|y_i(k) - x_i(k)|}{|y_i(k) - x_i(k)| + 0.5\max\max|y_i(k) - x_i(k)|} \tag{3}$$

where $|y_i(k) - x_i(k)|$ means the distance between $y(k)$ and $x(k)$. Due to min min $|y_i(k) - x_i(k)| = 0$ in the normalized case and 0.5 max max $|y_i(k) - x_i(k)|$ is a constant so that $\xi_i(k) \propto 1/|y_i(k) - x_i(k)|$. The smaller of $|y_i(k) - x_i(k)|$, the larger of $\xi_i(k)$.

$$\gamma_i = \frac{1}{n}\sum \xi(k) \tag{4}$$

where n is the number of the formulations.

2.4. Application of Golden Section to design friction material formulations

All components used in the formulations were divided into three groups: fibers (F), fillers (f) and binder (B) for non-metallic organic (NMO) or metals (M), non-metals (N) and binder for semi-metallic friction materials. Then the volume fraction of the components in each group can be calculated by Golden Section sequence of V_F (V_f or V_B) $\times$ $0.618^n = 0.236$, 0.146, 0.09, 0.056, 0.043, 0.034, 0.022, ... when $n = 1, 2, 3, 4, 5, \ldots$ (in the case of $V_F = 0.382$). Relational grade analysis was used to link the formulations and friction performance and give a direction of which ingredient its amount should be increased and which ingredient its amount should be decreased.

There are three phases in performing of Golden Section approach. Phase 1: V_F, and V_B were kept constant and all ingredients within fiber and filler group (F_1, $F_2, \ldots, f_1, f_2, \ldots$) were simultaneously changed. From the permutation of multiple components 10–12 formulations are selected as phase 1. Relational grade analysis was used to evaluate friction coefficient (μ) and wear of formulations in phase 1. From two ranks of the effects of each ingredient on μ and wear obtained by relational grade analysis, the fact can be known clearly that which component has better or bad effect in friction performance and the amount of that component in formulations can be adjusted; Phase 2: V_B was kept constant and V_F was proportionally increased (corresponding to V_f decreased) or decreased (V_f increased); Phase 3: V_B was varied (corresponding to $V_F + V_f$ varied).

3. Experimental

3.1. Raw materials

Raw materials used are listed in Table I. Seven raw materials were divided into three groups. Benzoxazine, a ring-open polymerized phenolic, was used as binder. Al_2O_3, $BaSO_4$, graphite and NBR (nitrile rubber) were

TABLE I Raw materials

Raw materials	Trademark	Company
Twaron pulp	1099	Akzo Noble Aramid Products
Slag fiber	PMF 204	Sloss Industries
Graphite	A 625	Asbury Carbons
Al_2O_3	Cement	Alfa Aesar
$BaSO_4$	290	Whittaker, Clark & Daniels
NBR	Nipol 1411C	Zeon Chemical
Benzoxazine	B	SCU

TABLE II Formulations in phase 1 selected from 7 components permutation

Vol%	BT-1	BT-2	BT-3	BT-4	BT-5	BT-6	BT-7	BT-8	BT-9	BT-10	BT-11	BT-12
Twaron	23.6	23.6	23.6	23.6	23.6	23.6	14.6	14.6	14.6	14.6	14.6	14.6
PMF	14.6	14.6	14.6	14.6	14.6	14.6	23.6	23.6	23.6	23.6	23.6	23.6
Graphite	23.6	9	14.6	9	9	5.6	23.6	9	14.6	5.6	14.6	9
Al_2O_3	9	2.2	9	14.6	9	9	9	3.4	9	14.6	5.6	9
$BaSO_4$	3.4	23.6	9	5.6	14.6	14.6	2.2	23.6	5.6	9	14.6	14.6
NBR	2.2	3.4	5.6	9	5.6	9	3.4	2.2	9	9	3.4	5.6
Benzoxazine	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6
μ	0.466	0.418	0.427	0.497	0.462	0.423	0.443	0.506	0.410	0.479	0.457	0.449
Wear%	12.74	8.32	15.00	7.87	12.86	11.85	19.33	30.34	20.12	17.62	24.08	22.57

selected as fillers group. Twaron (aramid pulp) and PMF (slag fiber) were chosen as fibers group.

3.2. Preparation of friction materials

Ingredients of Twaron, PMF, graphite, Al_2O_3 and $BaSO_4$ were dried at 110°C for 1 h and mixed them in a high speed blender for 40 s. Benzoxazine and NBR were milled and mixed together with other ingredients for 30 s. The mixture was molded under 210°C for 50 min and post-cured under 120°C for 90 min, 160°C for 90 min and 200°C for 60 min.

3.3. Friction performance tests

Friction performance (μ and wear) of small specimens with size of 13.2 mm × 12.7 mm × 4.5 mm was measured using Friction Assessment and Screening Test (FAST) under constant friction force mode. FAST was run at 6.96 m/s for 90 min with friction load 17.4 N. The wear was expressed as weight loss form as $W\% = (W_0 - W_1)/W_0 \times 100\%$, where W_0 and W_1 are the weight of the specimen before and after FAST respectively.

4. Results and discussion

4.1. Phase 1

The purpose of phase 1 is to find the optimizing proportion among V_{F1}, V_{F2}, V_{f1}, V_{f2}, V_{f3} and V_{f4} but keeps $V_F = 0.382$, $V_f = 0.382$ and $V_B = 0.236$. 12 formulations were selected from the all possible formulations (7 components permutation by Golden Section principle), as shown in Table II. The selection was randomly conducted.

TABLE III Relational grade analysis of BT1-12

γ_i	Twaron	PMF	Graphite	Al_2O_3	$BaSO_4$	NBR
For μ	0.9370	0.9038	0.8724	0.9234	0.6173	0.7007
For wear	0.8455	0.9399	0.8105	0.8602	0.6454	0.7170

Relational grade analysis was then used to evaluate the effects of each ingredient on μ and wear respectively, as shown in Table III.

Two ranks can be obtained from Table III as follow by simple expression if the middle position of two ranks (μ and wear) expressed as 0, the positive effect of ingredients on both μ and wear expressed as plus, and the negative effect of ingredients on both μ and wear expressed as minus, then following ranks were shown:

$$\begin{array}{ccccccccccccl} \text{Twaron} & > & Al_2O_3 & > & \text{PMF} & > & \text{Graphite} & > & \text{NBR} & > & BaSO_4 & \text{for } \mu \\ +2 & > & +1 & > & 0 & = & 0 & > & -1 & > & -2 & \end{array} \quad (5)$$

$$\begin{array}{ccccccccccccl} \text{PMF} & < & Al_2O_3 & < & \text{Twaron} & < & \text{Graphite} & < & \text{NBR} & < & BaSO_4 & \text{for wear} \\ -2 & < & -1 & < & 0 & = & 0 & < & +1 & < & +2 & \end{array} \quad (6)$$

Both μ and wear were affected by changing the volume fraction of the components. The component which shown the largest value of relational grade has the smallest distance to μ or wear. Due to the friction materials require stable μ, high μ and low wear, the component with large relational grade has more effect on μ than the component with small relational grade and the component with small relational grade has more effect on wear than the component with large relational grade.

According to the ranks the amount of PMF (0, −2) should be decreased because it has poor effect on wear and the amount of Twaron (+2, 0) should be increased because it has good effect on μ. From the results obtained by relational grade analysis, two formulations were designed based on BT-2 because of less amount of Al_2O_3 in BT-2 formulation, as shown in Table IV.

4.2. Phase 2

In phase 2 the optimizing proportion of V_F and V_f was searched. V_F will be proportionally changed either larger or smaller than 0.382 by $0.382 \pm 0.382 \times 0.618^n$ and V_f will be less when V_F is larger than or larger

TABLE IV Formulations designed by relational grade analysis

Vol%	Twaron	PMF	Graphite	Al_2O_3	$BaSO_4$	NBR	Binder	μ	W%
BT-13	32.6	5.6	9	2.2	23.6	3.4	23.6	0.445	4.66
BT-14	29.2	9	9	2.2	23.6	3.4	23.6	0.440	4.32

TABLE V Formulations in phase 2

Vol%	Twaron	PMF	Graphite	Al_2O_3	$BaSO_4$	NBR	Binder	μ	W%
BT-15	22.32	6.88	11.12	2.72	29.16	4.2	23.6	0.459	5.19
BT-16	36.08	11.12	6.88	1.68	18.04	2.6	23.6	0.452	6.82
BT-17	24.92	7.68	10.32	2.52	27.06	3.9	23.6	0.449	3.79

TABLE VI Formulations in phase 3

Vol%	Twaron	PMF	Graphite	Al_2O_3	$BaSO_4$	NBR	Binder	μ (89′)	W%
BT-18	26.04	8.03	10.78	2.63	28.28	4.08	20.16	0.449	3.81
BT-19	23.80	7.33	9.86	2.40	25.84	3.72	27.04	0.468	4.44

when V_F is less than 0.382. $V_B = 0.236$ keeps same as phase 1. Based on BT-14, three formulations were designed as phase 2 (Table V). The effect of $V_F(V_f)$ on wear is shown in Fig. 1.

4.3. Phase 3

To find the optimizing proportion between V_B and $(V_F + V_f)$, V_B will be changed either larger or less than 0.236 by $0.236 \pm 0.236 \times 0.618^n$. Based on BT-17, two formulations were used as phase 3 (Table VI). The effect of V_B on wear is shown in Fig. 2.

By doing 19 experiments, an optimizing formulation (BT-17) was obtained with stable μ compared with BT-2 and BT-14, as show in Fig. 3, average $\mu = 0.440$ and wear = 3.79 wt%.

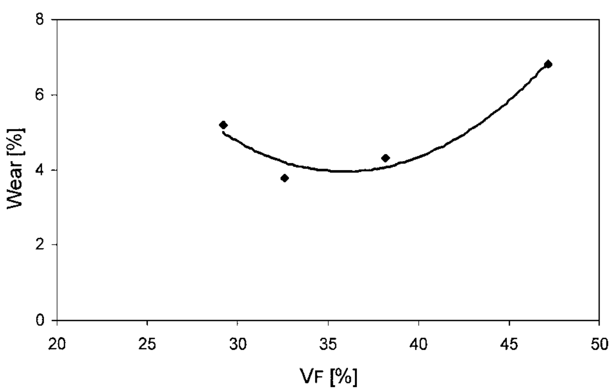

Figure 1 Effect of V_F (V_f) on wear.

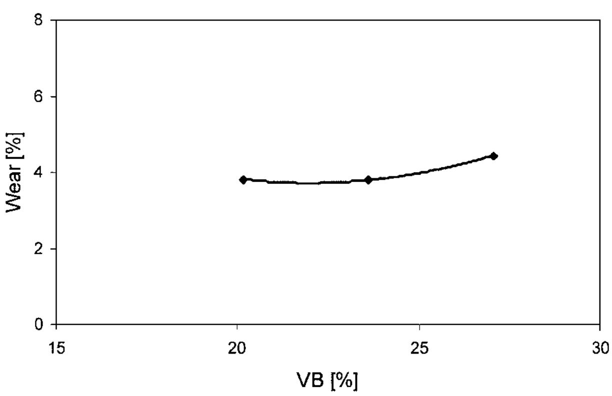

Figure 2 V_B on wear.

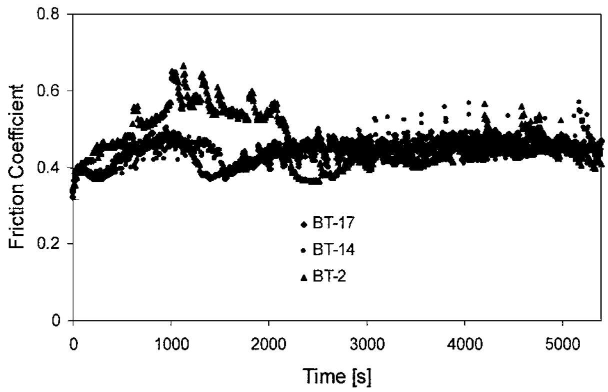

Figure 3 FAST curve of BT-17, BT-14 and BT-2.

5. Conclusions

The theoretical basis of optimizing formulation technique based on Golden Section and relational grade analysis was established by Golden Section sequence and components permutation governed by Golden Section. The optimizing formulation technique was tested using seven components including fibers group, fillers group and binder. The formulations were designed in three phases. Volume fraction of three groups was kept constant but each fiber and filler within their group were changed in phase 1. Volume fraction of binder was kept constant but increasing fibers and decreasing fillers or decreasing fibers and increasing fillers in phase 2. Volume fraction of binder was varied in phase 3. An optimizing non-metallic friction material formulation (BT-17) with stable μ and low wear was obtained after doing 19 experimental runs.

Acknowledgments

This work was performed at the Center for Advanced Friction Studies (CAFS), Southern Illinois University Carbondale, USA and the author thanks Dr. Maurice A. Wright, Director of CAFS, for his financial support.

References

1. J. BIJWE, *Polym. Compos.* **18** (1997) 378.
2. L. GUDMAND-HOYER, A. BACH, G. T. NIELSEN and P. MORGEN, *Wear* **232** (1999) 168.
3. G. CROSA and I. J. R. BAUMVOL, "Advances in Composite Tribology" (Elsevier Science, Amsterdam, 1993) p. 583.

4. C. F. TANG, Master Thesis (Southern Illinois University at Carbondale, 2002).
5. Y. LU, C. F. TANG and M. A. WRIGHT, in Proceedings of 2001 Colloquium on Friction Materials & Brake System (Southern Illinois University, May 2001, Carbondale, IL, 2001).
6. D. M. ELZEY, R. V. VANCHEESWARAN, S. W. MYERS and R. G. MCLELLAN, Proceedings of the 18th Annual Brake Colloquium & Engineering Display, San Diego, October 2000 (Society of Automotive Engineers, Warrendale, PA, 2000) p. 181.
7. R. ROBERT, M. C. FEMANDEZ and D. RIVACOBA, in Proceedings of the 18th Annual Brake Colloquium & Engineering Display, San Diego, October 2000 (Society of Automotive Engineers, Warrendale, PA, 2000) p. 17.
8. J. CHE, Y. SONG, Y. LU, W. MA and F. JI, *J. Univ. Nanjing Sci. Tech.* **23**(3) (1999) 16.
9. G. DRAVA, R. LEARDI, A. PORTESANI and E. SALES, *Chemom. Intell. Lab.* **32** (1996) 245.
10. T. KATO and H. SOUTOME, *Tribology Trans.* **44** (2000) 137.
11. W. CHENEY and D. KINCAID, "Numerical Mathematics and Computing" (Brooks/Cole Publishing, Pacific Grove, 1994) p. 494.
12. G. E. RUNION, "The Golden Section" (Dale Seymour, Palo Atlo, 1990) p. 21.
13. D. CHAPPELL and C. STRAKES, *The Mathematical Gazette* **74** (467) (1990) 43.
14. Y. LU and M. A. WRIGHT, in 45th International SAMPE Symposium and Exhibition (Proceedings), Long Beach, May 2000 (Society for Advancement of Material Process Engineering, Covina, CA, 2000) p. 306.
15. Z. PENG and T. B. KIRK, *Wear* **225–229** (1999) 1238.
16. S. LIU and Y. LIN, "An Introduction to Grey Systems: Foundations, Methodology and Applications" (IIGSS Academic Publisher, Slippery Rock, 1998) p. 63.
17. Y. LU and M. A. WRIGHT, in 45th International SAMPE Symposium and Exhibition (Proceedings), Long Beach, May 2000 (Society for Advancement of Material Process Engineering, Covina, CA, 2000) p. 1972.

Received 10 April 2001
and accepted 5 November 2002